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FAST TRACK COMMUNICATION

Far-field x-ray phase contrast imaging has no detailed information on the object

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Abstract
We show that x-ray phase contrast images of some objects with a small cross-section diameter d satisfy a condition for a far-field approximation $d \ll r_1$ where $r_1 = (\lambda z)^{1/2}$, λ is the x-ray wavelength, z is the distance from the object to the detector. In this case the size of the image does not match the size of the object contrary to the edge detection technique. Moreover, the structure of the central fringes of the image is universal, i.e. it is independent of the object cross-section structure. Therefore, these images have no detailed information on the object.

Widely used techniques of x-ray imaging are based on absorption because they do not require a coherent source. However, absorption of hard x-rays in soft materials is small, and it leads to a weak contrast. Meanwhile, the complex refractive index $n = 1 - \delta + i\beta$ has also a real part, whose difference from unity δ exceeds β by two orders or more [1, 2]. Therefore, a passage of plane wave through matter changes its amplitude by the factor $\exp(-2\pi\beta t/\lambda)$ where t is the matter thickness, λ is the wavelength of monochromatic radiation, and its phase by the value $-2\pi\delta t/\lambda$ compared with the case of free space.

The phase cannot be imaged just behind the object because any detector registers the intensity. However, the phase variation may be transformed to an intensity variation by means of some interference phenomenon [3]. Of course, this is possible only with a coherent illumination of the object and only at some distance from the object. It was first discovered in 1995 [4] that the sources of synchrotron radiation of third generation are just able to provide a necessary coherent illumination, so the x-ray phase contrast imaging becomes possible since 1995 as a widely used tool. We note that there are also three techniques which do not require synchrotron radiation [5–7], a full bibliography can be found in [2].

Here we will consider an approach based on synchrotron radiation (SR). In this case the experimental setup is very simple. A point source of divergent radiation is placed at large distance from the object. Since refraction of hard x-rays is very small we have large distances (centimetres or more) along the optical axis (z-axis) and rather small distances across the optical axis (x- and y-axes). The object is illuminated by spherical wave in a paraxial approximation which can be described by the Kirchhoff propagator

$$P(x, z) = (i\lambda z)^{-1/2} \exp(i\pi x^2/\lambda z) \qquad (1)$$

in a one-dimensional case. Very often the objects are rather thin and weakly reflect the x-rays. In this case they can be described by the transmission function

$$T(x) = \exp(2\pi i(n - 1)t(x)/\lambda). \qquad (2)$$

Similarly, in the case of visible light [3] one can distinguish three various regions behind the object where imaging the object is different.

The first region is called near field (small distances). In this region the size of the object greatly exceeds the size of the first Fresnel zone $(\lambda z)^{1/2}$. The third region is called far field (large distances). In this region the first Fresnel zone exceeds much the size of the object. The second region is just between those mentioned above. Since for hard x-rays the parameter λ is equal to 0.1 nm or less, the size of the first

Fresnel zone at a distance of 1 cm is equal to 1 μm. As a rule, the size of the object is more than 10 μm, so the near-field approximation is valid for distances up to 100 cm. There is a rather good method which allows one to reconstruct the phase profile from the intensity profile with the use of transport-of-intensity equation [8]. One can use as well the stationary phase approximation [9]. These methods work well for the objects with a smooth thickness profile $t(x)$, i.e. if the phase profile of the wave field has no sharp jumps.

However, many objects have a circular cross-section like a fibre and a rather large diameter of about 100 μm. For these objects $t(x)$ has sharp jumps at the boundaries. These boundaries well incline the rays out of the object where they interfere with the free space rays. This interference leads to a sharp peak of intensity even at very small distance from the object. Therefore many objects can be imaged just by its boundaries. This technique was called edge detection [10, 11]. We note that edge detection does not require a very high level of coherence. It allows one to image the object even with the synchrotron radiation source of second generation [12].

The goal of this paper is to show that there are some objects of small cross-section for which the methods of near-field approximation are not applicable. Moreover, the far-field approximation has to be used. Such objects exist in some crystals as crystal lattice defects: hollow core dislocations, or micropipes, in silicon carbide [13]; micro-channels in synthetic quartz [14], etc. Similar objects may exist as a part of a biological sample: a lung's airways and blood vessels. Despite the ability of phase contrast imaging and coherent-enhanced CT to provide high resolution images of lung airways and vessel morphologies [15, 16], quantitative evaluations of their diameters (<100 μm) are prohibited. For these objects the phase contrast image has a universal shape and this shape has no detailed information on the object.

As an example, we consider a silicon fibre of 1 μm diameter with the axis perpendicular to the x-ray beam direction. The energy of x-ray photons is 20 keV. The near-field image is too small for a registration by modern detectors. In addition, the edge detection is impossible because the maximum phase shift is 0.123, and the geometrical optics is not realized. At a distance of 1 m from the object the size of the image becomes larger. Of course, one prefers to choose this distance for the experiment.

The intensity profile $I(x)$ at the detector is calculated theoretically as a convolution of the transmission function with the Kirchhoff propagator modified by scaling factors which account for a divergence of the beam

$$I(x) = |a(x_0)|^2, \quad a(x_0) = \int \mathrm{d}x_1 P(x_0 - x_1, Z) T(x_1),$$
$$x_0 = x\frac{z_0}{z_t}, \quad Z = \frac{z_0 z_1}{z_t}, \quad z_t = z_0 + z_1, \tag{3}$$

where z_0 is the distance from the source to the object and z_1 is the distance from the object to the detector. The integral has to be calculated in the infinite limits while the integrand is a non-vanishing function at the limits. Therefore, it is convenient to use a transformation of the integral to the form

$$a(x_0) = 1 + \int \mathrm{d}x_1 P(x_0 - x_1, Z)[T(x_1) - 1]. \tag{4}$$

Such an integral shows a contrast directly and the integrand differs from zero only inside the object.

For our small object we can neglect absorption and expand the exponent of the transmission function into the Tailor series considering only the term of the first order. Then

$$T(x_1) = 1 + \mathrm{i}P\left(1 - \frac{x_1^2}{R^2}\right)^{1/2}, \qquad |x_1| < R,$$
$$T(x_1) = 1, \quad |x_1| > R, \qquad P = -\frac{4\pi\delta}{\lambda}R_0. \tag{5}$$

Here R, R_0 are the transverse and longitudinal radii of the elliptical cross-section of the fibre. On the other hand, we can explore a standard far-field approximation and neglect the term $\pi(x_1/r_1)^2$ in the phase of the propagator, where $r_1 = (\lambda Z)^{1/2}$ is the first Fresnel zone radius. As a result, we have

$$a(x) = 1 - \mathrm{i}^{1/2}\frac{2\pi^2\delta}{\lambda r_1}RR_0 \exp\left(\mathrm{i}\pi\frac{x^2}{r_1^2}\right)b(x), \tag{6}$$

where

$$b(x) = \frac{2}{\pi}\int_{-1}^{1}\mathrm{d}t \exp\left(-2\mathrm{i}\pi\frac{xR}{r_1^2}t\right)(1 - t^2)^{1/2}$$
$$= \frac{2x_c}{x}J_1\left(\frac{x}{x_c}\right), \qquad x_c = \frac{r_1^2}{2\pi R}. \tag{7}$$

Here $J_1(x)$ is the Bessel function of the first order. We note that $b(0) = 1$. Within the linear approximation over R, R_0 we obtain the following expression for the intensity:

$$|a(x)|^2 \approx 1 - \frac{4\pi^2\delta}{\lambda r_1}RR_0 b(x)\cos\left(\pi\frac{x^2}{r_1^2} + \frac{\pi}{4}\right). \tag{8}$$

This formula allows us to make some interesting conclusions. In the far-field region $x_c \gg r_1$. Then in the central part of the image, i.e. $|x| < x_c$, the structure of the image is determined by the cosine term, which is independent of the object. The object structure is described by the function $b(x)$ which modifies only far fringes. However, if the incident radiation is not monochromatic, i.e. it is described by some incoherent spectral function, then far fringes will be smoothened and only the central fringes will be visible which does not contain detailed information on the object. The only parameters, which can be determined, is a product RR_0 which influences a contrast level.

We consider below objects for which the problem mentioned above really exists, namely, micropipes in the SiC single crystal. Phase contrast images of the individual micropipes were obtained using white beam at a third-generation SR source, namely, Pohang Light Source, Pohang, Korea. The experimental setup does not contain a monochromator. However, the radiation intensity of high energy harmonics decreases sharply in the SR spectrum. On the other hand, the crystal itself forms the effective spectrum by means of absorption of soft x-ray radiation harmonics. We reveal that the effective x-ray spectrum of a high brilliance has a pronounced maximum at 16 keV, and it enables us to form partially coherent images even for transparent objects. Of course, only central fringes were visible. The crystal plate of thickness 490 μm was cut from the boule parallel to

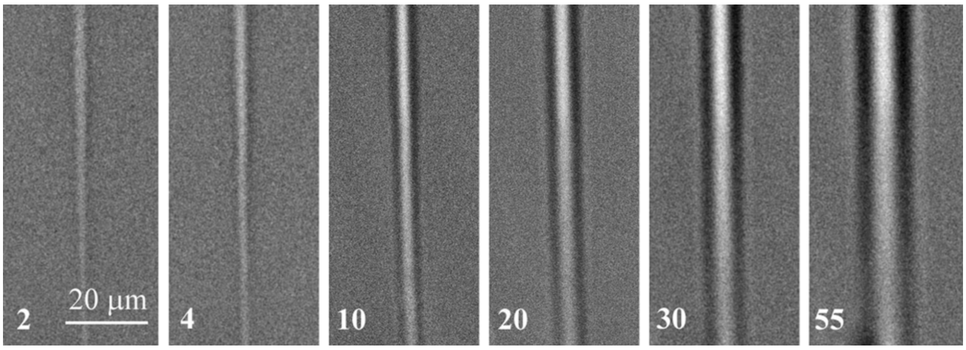

Figure 1. A series of images of the same micropipe in SiC crystal registered at various distances from the crystal which are shown on the panels by numbers in centimetres.

the growth direction and was placed normally to the beam. The images were recorded by high resolution CCD detector of 14-bit gray scale and 1600 × 1200 pixels range preceded by $CdWO_4$ fluorescent crystal to transfer x-rays to a visible light and optical system for a magnification of the images up to 50 times. The view field was 310 μm horizontally. Correspondingly the effective pixel size was 0.194 μm. The details of the experimental setup can be found in [17–19].

Figure 1 shows a series of images of the same micropipe obtained at various distances from the object from 2 to 55 cm. At a distance of 2 cm one can clearly see that a transverse cross-section diameter of the micropipe increases from bottom to top, i.e. it is inhomogeneous. However, the contrast is small. For long distances the contrast becomes more pronounced, but the structure of images is changed drastically. Now the transverse size of the images is constant from bottom to top, but the contrast level increases. This effect correlates completely with formula (8) where the minus sign must be replaced by a plus sign for a hole inside a matter. The images have a bright centre (i.e. the relative intensity is more than 1) followed by dark lines. The distance between dark lines is determined completely by the first Fresnel zone radius r_1 for a maximum intensity in the spectrum (16 keV).

The observation of this micropipe and many other similar micropipes provide strong evidence for the fact that their images have all properties of the far-field phase contrast described above. This understanding is new in the phase contrast physics. Up to now, many researchers have been keen on thinking about an edge detection and have tried to estimate the transverse size of the object through measuring the transverse size of the image. We would like to note that our conclusion is independent of the elliptical shape of a section of the object by the x-ray beam. The same conclusion may be done with any kind of section. In the arbitrary case the function $b(x)$ may have any shape, but the property $b(0) = 1$ and the parameter x_c remain unchanged. The structure of the cross-section cannot be revealed from the image. The only parameter which can be estimated is a size of cross-section area. Our conclusion is verified by other experimental results as well as computer simulations which will be presented in a more extended paper.

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