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Semianalytical Theory of Focusing Synchrotron Radiation by an Arbitrary System of Parabolic Refracting Lenses and the Problem of Nano-Focusing

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Abstract—It has been shown that an arbitrary system of parabolic refracting lenses does not change the shape of the Gaussian wave function of a synchrotron radiation beam. Only three parameters of the wave function change, for which the recurrent formulas are derived. These formulas allow one to perform quickly and accurately the calculation of optical properties of an arbitrary system of lenses. The parameters of the radiation beam calculated by the developed method have been compared to the results of the theory of continuously refracting lens. It has been shown that both approaches give surprisingly close values. The problem of focusing the beam to a nanometer size is discussed. A two-lens system has been proposed, which can provide a five-fold increase in the number of photons inside the focus, although we failed to decrease the beam size.

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1. INTRODUCTION

To develop nanotechnology along with many others, the realization of focusing the hard synchrotron radiation beam by different focusing systems is of great importance. The problem consists in the obtaining of a submicrometer and, in sight, a nanometer size of a beam. Hard X-rays have a wavelength λ smaller than 0.1 nm, however the problem is that it interacts very weakly with material and, in particular, is very weakly refracted with the refractive index $n = 1 - \delta$, where $\delta < 10^{-5}$. It is intuitively clear that it should result in the specified limitations on the minimal size of the focused beam. The theoretical limit $\lambda/(8\delta)^{1/2}$ on a size of the beam formed by the waveguide technique using the effect of total external reflection from a smooth surface of the material was determined in [1]. The reason is the finite value of the critical angle of total external reflection. So, for Si at λ = 0.1 nm we have $\delta = 0.31 \times 10^{-5}$ and $\lambda/(8\delta)^{1/2}$ = 20 nm.

As is known, when the Fresnel zone plates are used, the size of the beam is approximately the size of the boundary zone. To provide a necessary shift of a phase in zones, a zone thickness cannot be smaller than several micrometers. As a result, boundary zones have the form of long narrow channels. Thorough theoretical calculation of these zone plates was carried out in [2, 3], where it was shown that the focusing of the beam to a smaller size than the above indicated one is possible, although a portion of radiation does remain in the channels of boundary zones and does not go through the zone plate.

As for compound refracting lenses, the development of which began rather recently [4], it was shown in [5] that they also allow one to overcome the indicated above limit, in particular, it is possible to obtain the beam with a size up to 2 nm, although it is related to the solution of complex technological problems to produce the lenses with a very small size of surface curvature. An adiabatically focusing lens has a set of elements, the aperture of which decreases with a decrease in the beam size was proposed in [5]. Along with it, the analysis of properties of this lens and even its construction represented in [5], there remain many questions without answers. The result was virtually obtained by numerical calculations using the procedure a precision of which was not estimated.

To analyze the properties of compound refracting lenses with a parabolic surface profile, analytical methods of the solution of the problem of radiation transport through a refracting medium and in air are rather useful. The exact (analytical) solution of the problem for a parabolic continuously refracting lens which composes of a very large number of identical elements with a large radius of surface curvature was obtained in [6, 7]. Each element refracts very weakly that allows one to average its density over a thickness of the element and, as a result, to consider a medium being uniform along the beam with a parabolic change in density in the transverse direction.

This lens has no aberration even under the condition that its length is comparable to its focal distance [8], in contrast to a one-fold lens with the same length and focal distance. However, the analysis has shown that the lens composed of identical elements is not effective for a length comparable to a focal distance, since its last elements very weakly effect the beam. To focus more effectively it is necessary to use a compound lens having different elements [5, 9], which take into account the change in the transverse sizes of the beam.

In the present work, the semianalytical theory of focusing synchrotron radiation by an arbitrary system of parabolic refracting lenses is presented. It has been shown that the shape of the wave function does not change when a coherent wave described by the wave function of the specified Gaussian type with three parameters propagates through an optical system consisting of parabolic refracting lenses arranged along an optical axis at arbitrary distances, only the numerical values of the parameters change. Recurrent formulas allowing the quick and accurate calculation of the parameters of the synchrotron radiation beam after focusing of this lens system have been obtained for the latter.

In the next section the recurrent formulas are derived for the parameters of the wave function of coherent radiation, and general properties of a beam are discussed based on the analysis of indicated parameters. The comparison of the given theory with the theory of continuously refracting lens [6, 7] is presented in the third section, and the dependence of the main parameters of lenses on a number of its elements is analyzed. Then the problems of focusing the beam to a nanometer size are discussed. Some results of the analysis of optical properties of a two-lens system are presented in the last section.

2. BASIC FORMULAS OF THE THEORY

As is known, the synchrotron radiation is to a large degree polarized and, in addition, the polarization does not affect the result of focusing, therefore it is sufficient to consider the scalar wave function of radiation $\Psi_2(x, y, z)$. Let the z axis of the coordinate system be directed along an optical axis and correspond to large distances (from millimeter to several meters) and the x and y axes be directed perpendicularly to an optical axis, the characteristic scale of the change in the wave function in these directions is tens of micrometers. Since the angles of scattering for X-rays are very small, it is possible to a sufficient degree of accuracy to use the paraxial approximation in which a spherical wave is replaced by a cylindrical one. When the radiation is transferred through empty space, the change in the wave function is described by its convolution with the Kirchhoff propagator [10]

$$P_2(x, y, z) = P(x, z)P(y, z),$$
$$P(x, z) = (i\lambda z)^{-1/2}\exp(i\pi x^2/\lambda z), \quad (1)$$

where λ is the wavelength of monochromatic radiation.

Consider thin biconcave parabolic lenses which do not change the transverse coordinate of X-ray trajectory over the entire thickness. Their influence on a wave function is reduced to a phase shift locally in each point including its imaginary part related to the absorption. This change is described by the method of multiplication of the wave function of radiation by the transparent function, which is standard in the method of phase contrast,

$$T_2(x, y, f_c) = T(x, f_c)T(y, f_c),$$
$$T(x, f) = \exp(-i\pi x^2/\lambda f), \quad f_c = \frac{R}{2(\delta - i\beta)}, \quad (2)$$

where R is the radius of the curvature of the parabolic profile of the lens and the parameters δ and β are related to the dielectric susceptibility ε of the lens material by the formula $\varepsilon = 1 - 2(\delta - i\beta)$. The parameter f_c is the complex focal length of a lens. In general, expression (2) is applied only inside the geometric lens aperture and out of it the function is constant. We shall restrict our consideration only to such beams of transverse sizes, of which are smaller than the geometric aperture, therefore, its accounting weakly affects the result and this influence can be neglected. If this is not the case, it is sufficient to modify a wave function at the output in such a way that it's half-width is the geometric aperture, since a part of the wave function beyond the aperture in any case is out the focus.

The focusing of a point source by a system of these lenses separated along an optical axis at some distances is of our interest. Eqs. (1) and (2) define the functions with a cylindrical symmetry, at which the coordinates x and y are described independently. Apparently, the radiation wave function will also have the identical symmetry, i.e., $\Psi_2(x, y, z) = \Psi(x, z)\Psi(y, z)$. In this case it is sufficient to restrict the consideration to the analysis in any one plane, for example (x, z), since in the other plane everything will be analogous. Then we will not use the z coordinate of the wave function and, instead, use the index indicating how many parabolic lenses were operated. On the other hand, we will explicitly take into account the transverse coordinate x_0 of the point source.

So, if we know the wave function $\Psi_0(x, x_0)$ in front of a parabolic lens with a complex focal length f_c, then the wave function beyond a lens at the distance z from it will be described by the integral

$$\Psi_1(x, x_0) = \int dx_1 P(x - x_1, z)T(x_1, f_c)\Psi_0(x_1, x_0). \quad (3)$$

Formulate the theorem: if the incident wave function $\Psi_0(x, x_0)$ has a Gaussian shape in the form

$$\Psi_0(x, x_0) = T(x, a_0)P(x - x_0, b_0)T(x_0, c_0) \quad (4)$$

with the complex parameters a_0, b_0, and c_0, then the wave function $\Psi_1(x, x_0)$ determined by equation (3) has the same shape

$$\Psi_1(x, x_0) = T(x, a)P(x - x_0, b)T(x_0, c), \quad (5)$$

in which only the parameters a, b, and c take the other values. The new parameters are determined by the relations

$$a = d\frac{b}{b_0}, \quad b = b_0 + z\left(1 - \frac{b_0}{d}\right), \quad c = \frac{c_0}{1 + zc_0/bd}, \quad d = \frac{a_0}{1 + a_0/f_c}. \tag{6}$$

using the old ones.

The proving of the theorem can be obtained by the transformation of integral (3) to the tabulated integral

$$\int dx \exp(-i\beta x + i\gamma x^2) = \left(\frac{i\pi}{\gamma}\right)^{1/2} \exp\left(-i\frac{\beta^2}{4\gamma}\right). \tag{7}$$

Note that the right multiplier in (4) and (5) does not define the transverse structure of the wave function, i.e., does not depend on the x coordinate. In the pure state, it describes the dependence on the source coordinate. For this reason, the parameters a and b do not depend on c_0, but c depends on all three parameters a_0, b_0, and c_0.

Consider particular cases. If $z = 0$ (just beyond a lens), then $b = b_0$, $c = c_0$ and only the parameter a changes and testifies the relation $a^{-1} = a_0^{-1} + f_c^{-1}$. It immediately follows from (3), if the Kirchhoff propagator is replaced by the delta function. If $a_0^{-1} = f_c^{-1}$, then $a^{-1} = 0$, $c = c_0$, and $b = b_0 + z$. These relations immediately follow from the properties of the Kirchhoff propagator [10]. In general, recurrent relations (6) have rather a complex and not quite visual view. Nevertheless, they allow us to calculate very quickly the wave function for any arbitrarily complex system of parabolic lenses. Consider the simplest situation with one lens as an example. Let the distance between a radiation source and a lens be z_0. The wave function at the distance z_1 beyond a lens is of interest. Assuming $a_0^{-1} = c_0^{-1} = 0$, $b_0 = z_0$, and using Eq. (6) we obtain

$$a_1 = \frac{f_c}{z_0}b_1, \quad b_1 = z_0 + z_1 - \frac{z_0 z_1}{f_c}, \quad c_1 = \frac{f_c}{z_1}b_1. \tag{8}$$

This result was previously obtained in [7].

Suppose that we took into account everything in the parabolic lenses of the investigated system and obtained the wave function $\Psi_n(x, x_0)$ with the parameters a, b, and c. The radiation intensity is experimentally measured, i.e., square of the wave function module. We are interested in the dependence of the intensity $I(x, x_0, z)$ on x in the process of variation in the distance z between a detector and the last nth lens in the system is. It is convenient to calculate the relative intensity dividing it by the source radiation intensity at the distance z between it and the last lens. From wave function (5) for the intensity, we have the expression

$$\begin{aligned} I(x, x_0, z) &= |\Psi_n(x, x_0)|^2 \\ &= I_m(z, x_0)\exp\left(-\frac{(x - x_m(z))^2}{2\sigma^2(z)}\right), \\ I_m(z, x_0) &= \frac{Z}{|b|}\exp\left(-\frac{x_0^2}{2\sigma_0^2}\right). \end{aligned} \tag{9}$$

Here, we introduce the following parameters:

$$\begin{aligned} \sigma(z) &= \left(\frac{4\pi}{\lambda}[A - B]\right)^{-1/2}, \quad x_m(z) = -Mx_0, \\ M &= \frac{B}{A - B}, \quad \sigma_0 = \left(\frac{4\pi}{\lambda}[C - AM]\right)^{-1/2}, \end{aligned} \tag{10}$$

where

$$A = |a|^{-2}\mathrm{Im}(a), \quad B = |b|^{-2}\mathrm{Im}(b), \quad C = |c|^{-2}\mathrm{Im}(c). \tag{11}$$

It follows from (9) that the dependence of the intensity on the x coordinate is described by the Gaussian function for all of z, but the parameters of the Gaussian curve like the half-width $w(z) = C_w\sigma(z)$, the position $x_m(z)$, and the height $I_m(z)$ change. Here and below, the constant is $C_w = (8\ln 2)^{1/2} = 2.3548$.

Taking into consideration some evident physical properties for a focused beam, one can make the general conclusions relative to the parameters. So, the peak height depends on the z coordinate (through the b parameter) and on the source coordinate x_0. In this case, the dependence on x_0 reflects the fact that the beam trajectories for the points on a source strongly deviated from an optical axis can not pass through apertures of all lenses through the center. Since the lenses absorb, this results in a decrease in the intensity maximum. By figuratively speaking, the extended lens system defines the efficiency of different points on a source in the form of a Gaussian function with the half-width $w_0 = C_w\sigma_0$. This efficiency cannot obviously depend on z. We obtain from here that the quantity $(C - AM)$ does not depend on z. This property is especially valid in numerical calculations but it is very difficult to prove it analytically for the system composing of a large number of lenses.

Then the radiation intensity being integral with respect to the x coordinate is determined by the formula $S(x_0) = 1.0645w(z)I_m(z, x_0)$ where the constant is $((2\pi)^{1/2}/C_w)$. It is clear from the energy considerations that it also does not depend on z, since we neglect the radiation absorption in air. Substituting formulas we obtain for a source on an optical axis

$$A_\gamma = S(0) = \frac{\lambda^{1/2}Z}{[2|b|^2(A - B)]^{1/2}}. \tag{12}$$

This parameter will be further named an effective aperture. It shows what fraction of the total beam intensity passed through a lens system and will be focused. If lenses do not absorb, this fraction was determined just by an aperture. It follows from (12) that the quantity $G = |b|^2(A - B)$ does not depend on z. From this conclusion, we immediately obtain that $M = \mathrm{Im}(b)/G$ depends on z linearly. It is convenient to write this dependence in the form $M = (z + Z_1)/(z_0 + Z_0)$ with two parameters Z_0 and Z_1, where z_0 is the distance between a source and the first lens. Apparently, for one lens we have $Z_0 = Z_1 = 0$. However for the system consisting of two or more lenses these parameters already are not zero and can be negative.

The formula for a beam half-width can be written in the form $w(z) = C_w(\lambda/(4\pi G))^{1/2}|b|$, from which it follows that the dependence on z is determined only by the last multiplier $|b|$. As follows from formulas (6), $b = B_0 + zB_1$ is the complex parameter depending linearly on z. The complex parameters B_0 and B_1 can be determined, for example, from recurrent formula (6) taking into account the last lens. The distance at which a beam has minimal sizes, i.e., function $|b|^2$ has a minimum, will be named the distance of focusing z_f. From this condition, we immediately obtain $z_f = -\mathrm{Re}(B_0 B_1^*)/|B_1|^2$. The curve of the intensity transverse distribution apparently has the maximum height at the same point.

If we apply the obtained formula to the case of one lens, then we obtain $z_f = f/F$, where $F = 1 - f/z_0 + \gamma^2(1 - f/z_0)^{-1}$, $\gamma = \beta/\delta$, and f is the real part of the complex focal distance, i.e., without accounting absorption. It follows from here that the formula coincides with the well-known expression, if absorption is absent. The absorption can be neglected when a distant source is focused by a short-focus lens when $f/z_0 \ll 1$, because $\gamma^2 \ll 1$. However, when a lens is used as a collimator, the correction related to the absorption allows us to take into account the fine effects related to an aperture of a finite lens.

3. COMPARISON WITH THE THEORY OF A CONTINUOUSLY REFRACTING LENS

First of all, it is interesting to compare the results obtained by the above approach to the results of the theory of a parabolic continuously refracting (PCR) lens developed in [6, 7]. A lens composing of N identical biconcave elements with a thickness p is considered in the theory of PCR lens. The total lens length is $L = pN$. To obtain the analytical solution, the optical density of the substance in each element is averaged over its thickness, which results in the continuously refracting medium. The solution of the Maxwell equation for this medium gives the expression for the image propagator by a lens which exactly coincides with expression (5), but with the difference that the parameters are determined by analytical formulas, namely

$$a = b/g_0, \quad b = (r_0 + r_1)c_L + \left(z_c - \frac{r_0 r_1}{z_c}\right)s_L,$$
$$c = b/g_1, \quad g_{0,1} = 1 - c_L + \frac{r_{0,1}}{z_c}s_L, \tag{13}$$

where r_0 is the distance from a source to the origin of the lens, r_1 is the distance from the end of lens to a detector, and

$$s_L = \sin(L/z_c), \quad c_L = \cos(L/z_c),$$
$$z_c = (pf_c)^{1/2}. \tag{14}$$

To compare the PCR approximation to the approach developed above, in which each lens, is vise versa, is considered to be very thin, it is necessary to consider the system composing of N identical lenses with the distance p between them. In this case, the distance from a source to the first lens is $z_0 = r_0 + p/2$ and the distance from the last lens to a detector is $z = r_1 + p/2$. Selecting the starting parameters $a_0^{-1} = c_0^{-1} = 0$, $b_0 = z_0$ and using recurrent relations (6) N times we obtain the answer. For numerical calculations the parameters of real lenses are used [11, 12], namely, the material is aluminum; the curvature of the parabolic surface of elements, $R = 0.2$ mm; the thickness of elements, $p = 1$ mm; the photon energy, $E = 25$ keV; and the distance, $r_0 = 50$ m. The most interesting parameters are the focal distance z_f, the half-width of a beam in the focus $w(z_f)$, and the effective aperture A_γ. We consider the case when a real aperture is larger than an effective one, therefore both approaches are valid.

For calculation, the ACL programs [13] were used for an interpreter, written in Java [14]. The a, b, and c parameters were calculated by two methods and the parameters indicated above were calculated in the N range from 30 to 520 using them. Note that a lens focuses at its end for $N = 527$, i.e., $z_f = 0$ and the considered above method for the determination of a focal distance is not applied when N increases. It turned out that both methods result in the surprisingly close results for all N for all three parameters, and the difference decreases with an increase in N, which is also surprising. A relative difference has the order of magnitude of 10^{-6} for a focal distance and integral intensity and 10^{-7} for the half-width of a beam in the focus.

The curves of the dependence of the parameters z_f, $w(z_f)$ and A_γ on N are shown in Fig. 1. As one can see, all three parameters change rather quickly at small values of $N < 200$, i.e., a distance of focusing and a half-width of a beam in the focus decrease. The effective aperture decreases simultaneously. However for $N > 200$ these dependences become weaker. A decrease in the distance of focusing is described by an almost linear curve with a relatively small slope and a decrease in the half-width of the beam and the aperture virtually stops. It is also inter-

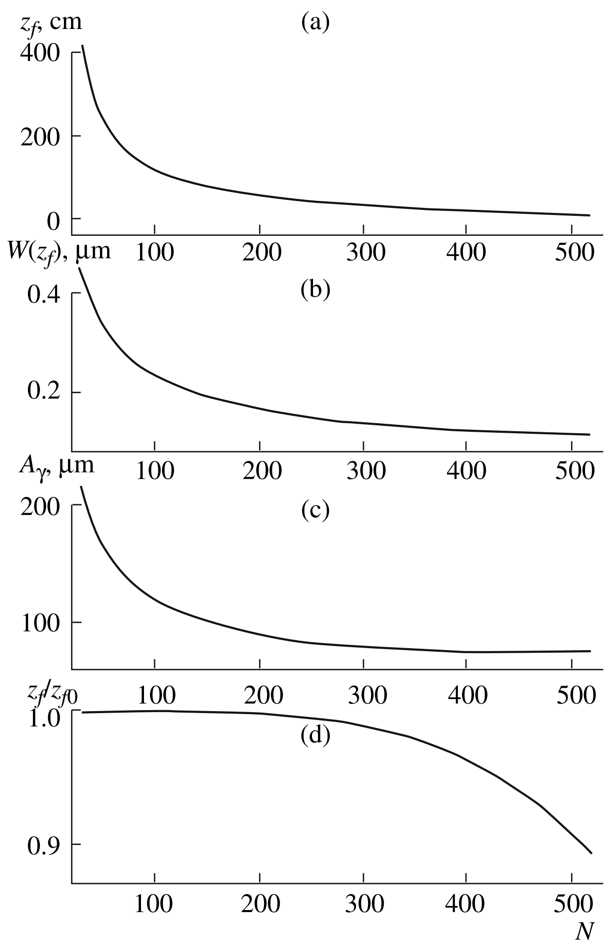

Fig. 1. Dependence of the parameters of a refracting lens composed of a set of identical elements on a number of these elements$N$: z_f is a focal distance counted from the lens end (a); $w(z_f)$ is a focus half-width (b); A_γ is an effective aperture (c); z_f is a focal distance counted from the center of lens, z_{f0} is the estimation of a focal distance in the approximation of a thin lens with correction (d).

esting to compare the distance of focusing of a long lens with the formula taking into account the correction to the value for a thin lens. The latter was derived in [7, 15, 16]. The correction to the distance of focusing takes into account the lens length L. The distance of focusing is counted from the midpoint of the lens and is $f_0 = R/(2N\delta) + L/6$ for a parallel beam. The distance of the source image is calculated by the formula for a thin lens $z_{f0} = f_0/(1 - f_0/(r_0 + L/2))$. The dependence of z_f/z_{f0} on N, where $z_f = z_f + L/2$, is shown in Fig. 1. It is seen that for $N < 200$, this relation virtually is at unity and then it decreases only by 10%. The decrease is explained by the fact that the correction L^2 in the expansion of a focal distance in the terms of L has a negative sign. It is interesting that even for the lens focusing at its end, i.e., when $z_f = L/2$, the focal distance slightly exceeds the exact value in the approximation of a thin lens with the correction.

The calculation results have shown that it is profitable to compose a lens of a large number of identical elements only up to those limits when it can be considered to be thin, i.e., its length is smaller than the focal distance. Otherwise, a beam contracts inside the lens and the additional elements with a large radius of curvature are not effective.

4. ABOUT FOCUSING TO NANOMETER SIZE

Consider one thin absorbing lens focusing a parallel beam, i.e. in the limit $z_0 \longrightarrow \infty$. Using formulas (8)–(12) in the approximation linear with respect to the parameter $\gamma = \beta/\delta$ and performing the limiting transition, we obtain that the focal distance $f = R/(2\delta)$, the half-width of a beam in the focus $w(f) = 0.6643(\lambda f\gamma)^{1/2}$, where the constant is $C_w/(4\pi)^{1/2}$, and the effective aperture $A_\gamma = C_s w(f)/\gamma$, where $C_s = 1.0645 = (2\pi)^{1/2}/C_w$. It follows from the latter formula that irrespective of the focal distance of a lens, the radius of its curvature and other parameters, the half-width of a beam in the focus is always γ times smaller than that of a beam just beyond the lens $w(0) = A_\gamma/C_s$. By the way, this rule is well performed in the calculation results presented in Fig. 1. The ratio $w(z_f)/A_\gamma$ monotonously decreases with an increase in N from 0.002 to 0.0015, whereas $\gamma = 0.002$. Note also that the relation $w(f) = 0.4697\lambda f/A_\gamma$ is fulfilled, where the coefficient is $1/(2C_s)$, i.e., in the considered case, the focus size is almost two times smaller than in the case of a nonabsorbing lens, when $w(f) = \lambda f/A$.

Substituting the formula for the focal distance to the formula for a half-width of a beam in the focus, we obtain the condition for the radius of the curvature at which the focusing to the specified size w_1 is possible, namely $R_1 = 4.532 w_1^2 \delta/(\lambda\gamma)$, where the constant is $8\pi/C_w^2$. . It follows from this formula that it is a very difficult technical problem to obtain a beam with small sizes. As the beam size decreases twice, the radius of the curvature should decrease four times. A real lens has a finite length L and geometric aperture A related to this length. If the thickness of the material on an optical axis is neglected, then $A = 2(RL)^{1/2}$. According to our approximation the lens length should be smaller than a focal distance. Consider the limiting case $L_1 = f_1 = R_1/(2\delta)$. Then the geometrical aperture is $A_1 = R_1(2/\delta)^{1/2}$ and the effective aperture is $A_\gamma = C_s w_1/\gamma$. For validity of the used formulas, the condition $A_\gamma < A_1/2$, which can be rewritten in the form $w_1 > w_c = 0.3321\lambda/\delta^{1/2}$, should be fulfilled.

The last condition shows that a thin lens that has an aperture determined by absorption cannot focus a parallel beam to the size smaller than some specified size w_c. The obtained limiting size does not depend on the wave length and almost exactly is the parameter $W_c = \lambda/(8\delta)^{1/2}$ introduced in [1], as the universal lower limit for the X-ray beam size. It is interesting that it also does not depend on the absorption coefficient of the material and depends only on its refracting properties. For

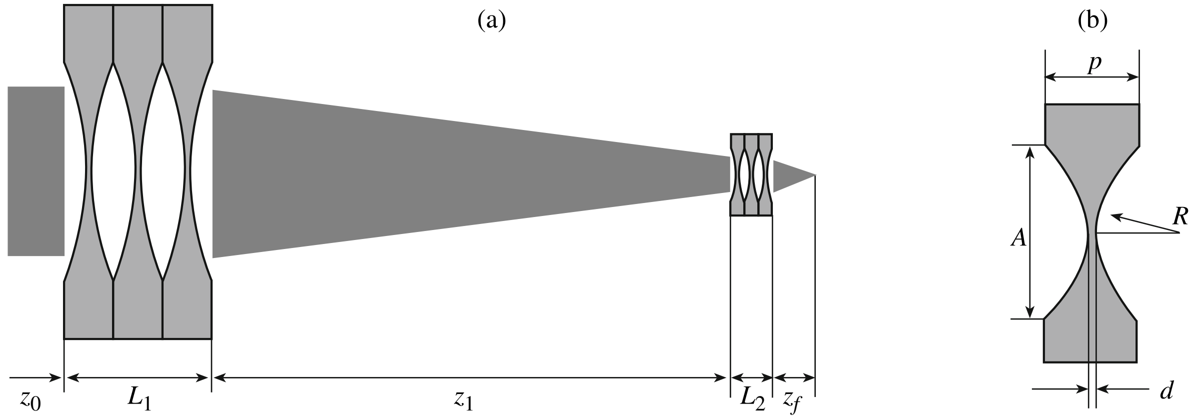

Fig. 2. Scheme of a two-lens system (a); parameters of one element (b).

example, $w_c = 15.2$ nm for diamond, 16.8 nm for aluminum, 18.6 nm for silicon, 22.3 nm for beryllium, and 42.4 nm for lithium. Dense materials have the smallest values, for example $w_c = 9.8$ nm for nickel and 7.3 nm for tungsten. For the limiting focus size, w_c the radius of the curvature of a lens is $R_c = \lambda/(2\gamma)$ and the focal distance is $f_c = \lambda/(4\beta)$. As for an effective aperture, it is only $1/\gamma$ times greater than the focus size. These parameters already depend on the radiation wave length and its absorption in material.

It is interesting to compare this result to the other limiting case of a very thin lens when the absorption can be neglected over all the lens length L. Choosing the radius of curvature from the condition $L = f$ for the geometric aperture A we obtain the same formula as above, i.e., $A = R(2/\delta)^{1/2}$ and the half-width of a beam in the focus is determined by the formula $w(f) = \lambda f/A = W_c$. In the given case the result does not depend on the radius of the curvature, since the focal distance is proportional to the aperture A. However, the radius of the curvature should not be large because the absorption will substantially affect the aperture. On the other hand, aberrations appear when a focal distance is of the order of the lens length [8]. If a compound lens is used, then the focal distance increases. One can assume that the situation drastically changes for the intermediate case when geometric and effective apertures have close values.

These conclusions are also valid for a compound lens in the form of a closely packed set of identical elements under condition that its length is smaller than the distance of focusing. As follows from the performed analysis, the focusing of a beam with extremely small sizes is realized by a lens with a very small aperture that results in a small number of photons in the focus. From the viewpoint of an increase in a number of photons in the focus and a decrease in the beam size, an adiabatic lens was considered in [5], but currently the preparation of these lenses is a virtually unattainable problem. A two-lens system in which the first lens has a large aperture and compresses a beam to a small aperture of the second lens is more real (Fig. 2, the left fragment). In this case, both lenses are composed of identical elements and operate as thin absorbing lenses. The number of elements in each lens can be relatively small and it means that the parasitic absorption in material on an optical axis will be minimal, whereas the beam compression beyond each lens occurs in air where the absorption is small.

5. ANALYSIS OF A TWO-LENS SYSTEM

To analyze numerically the optical properties of a two-lens-system in which each lens is composed of a random number of identical elements, the universal computer program based on recurrent formulas (6) has been developed. Although a zero thickness of elements is assumed in these formulas, a real lens has the specified length L, the thickness of one element is $p = L/N$, where N is a number of elements, and the geometric aperture is $A = 2(R(p-d))^{1/2}$, where d is a thin part of one biconcave element (Fig. 2, the right fragment). A geometric aperture specifies the maximum size of a beam which can really be focused. In the framework of the considered model it can be taken into account by the introduction of the new element into the origin of each lens which, not changing the wave phase, limits the amplitude with respect to the Gaussian law with the half-width equal to the geometric aperture. This procedure does not affect the result, if the effective lens aperture or the beam size is smaller than a geometric aperture. However if these conditions are not performed, then its use leads to the model which more correctly reflects a real lens.

The calculation results using the program can be formulated by the following. The size of the beam focused by the two-lens system cannot be smaller than that with one (second) lens. The beam size either remains or increases depending on the distance between lenses. At the same time, the intensity in the focus can increase several times. The maximally high intensity is obtained at such a distance between lenses, at which the beam width in front of the second lens

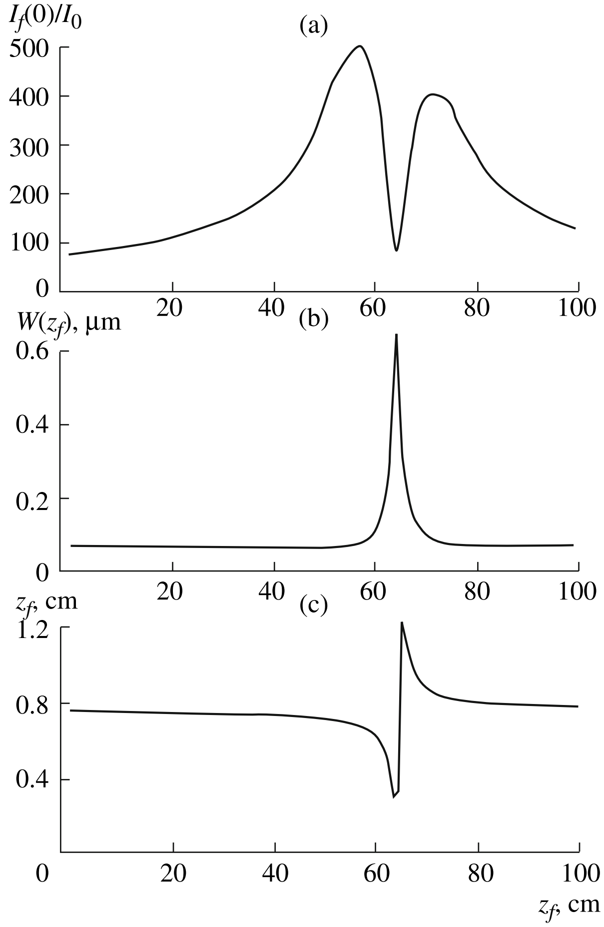

Fig. 3. Parameters of the X-ray beam focused by a two-lens system depending on the distance between lenses. Ratio of the peak intensity to that when a two-lens system is absent (a); the half-width of a beam in the focus (b); a focal distance counted from the end of the second lens (c).

coincides with its effective aperture. It takes place at the distances both smaller and larger than the focal distance of the first lens.

As an example, Fig. 3 illustrates the calculation results for the system which can really be performed in the planar variant. Both the lenses are made of silicon; the parameters of the first lens are $N = 25$, $L_1 = 1$ cm, $R = 100$ µm, and $d = 2$ µm and the parameters of the second lens are $N = 100$, $L_2 = 0.1$ cm, $R = 5$ µm, $d = 1$ µm, and the radiation wavelength is $\lambda = 0.1$ nm. In this case, the first lens has the effective aperture $A_\gamma = 55.5$ µm, $d = 1$ µm, and the radiation wavelength is $\lambda = 0.1$ nmm and the focal distance $z_f = 63.6$ cm. The maximum of relative intensity in the focus is $I_f(0)/I_0$ and the focus half-width is $w(z_f) = 543$ nm. The second lens has the effective aperture $A_\gamma = 6.0$ µm and the focal distance $z_f = 0.72$ cm. The maximum of relative intensity in the focus is $I_f(0)/I_0$ and the focus half-width is $w(z_f) = 65$ nm. Here and below, the focal distances are counted from the end of the compound lens in the form of a block composed of identical elements in the same manner as the distance between lenses z_1 (Fig. 2).

As is seen in the figure, the second lens becomes ineffective, if it is located in the focus of the first lens. On the other hand, if it is far from the focus of the first lens, then the beam size and the focal distance virtually do not depend on the first lens. Only the intensity in the focus monotonously changes depending on the distance between lenses z_1. Thus, a gain in intensity can be obtained in a wide distance range which is convenient for practical tuning of a two-lens system.

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