Hard x-ray focusing with extremely long compound refractive lens

Irina I. Snigireva [*a], Victor G. Kohn [b], Anatoly A. Snigirev [a]

[a] ESRF, B.P. 220, 38043 Grenoble, France;
[b] Russian Research Centre "Kurchatov Institute", 123182 Moscow, Russia

ABSTRACT

We present results of study of optical properties of extremely thick refractive lens when the thickness (or length) of a compound refractive lens is comparable with its focal distance. We tested a 2D parabolic compound refractive lens composed from 300-500 elements. Each element is a bi-concave lens made from aluminum with the curvature radius R = 0.2 mm and thickness 1 mm. Special long holders were designed and manufactured to keep up to 500 of elements. As far as the thin lens approximation is not valid we developed and used accurate theory of long parabolic compound lens for ray-tracing analysis. The experimental measurements were performed for the X-ray energies E = 20-30 keV. The measured focus distance and effective aperture correlate with the theory.

1. INTRODUCTION

Since the first successful observation [1] of synchrotron x-ray beam focusing with compound refractive lens (CRL), the x-ray refractive optics was developed rapidly. Different kinds of the compound lenses were proposed and tested, including drilled holes inside the block of matter [1,2], micro-bubbles embedded in epoxy [3], multi-prism or "alligator" x-ray lens [4], planar lens [5,6]. However, for the purpose of perfect imaging and 2D focusing the best solution is the 2D parabolic compound refractive lens. The first parabolic CRL was managed from aluminum [7-10] and was optimized for high-energy synchrotron beam. Today similar lenses were fabricated from beryllium [11] and even from lithium [12], which are more effective in the energy range from 10 keV to 20 keV.

The standard elementary aluminum lens is the bi-concave lens with the curvature radius R = 0.2 mm. The focal length F of such lens can be estimated within thin lens paraxial approximation as $F = R/2\delta$ where δ is the decrement of refraction index. At the photon energy E = 30 keV we obtain F = 167 m. Such long focal length is unacceptable. However, compound lens from 100 elements can be used successfully because for such CRL the focal length is 1.67 m. In principle, there is no limitation on the number of elements. However, several problems must be solved. First of all, each elementary lens has a finite thickness of 1 mm. Therefore CRL from 100 elements increases its thickness up to 10 cm, and for convenience we call it length instead of thickness. On the other hand, the effective aperture of such a lens decreases up to 0.2 mm. It is clear that the holder must provide a perfect alignment of such a CRL. Secondly, each elementary lens has a thin part where radiation intensity decreases due to absorption without focusing.

Finally, for very large number of elements the thin lens approximation is not valid and more accurate theory has to be developed. One can make ray-tracing [13,14] analysis but, this solution is not convenient because does not allow fast estimations of the main lens parameters. Fortunately, the propagator for the arbitrary long parabolic CRL can be derived in analytical form [15] similar to the free space Kirchhoff propagator. This allows one to develop diffraction theory of x-ray imaging with parabolic CRL [16,17] and to obtain many of the lens parameters in an analytical form. As it follows from the theory, the parabolic CRL with the arbitrary number of elements can accurately focus the x-ray beam without aberrations. For very many elements the focusing can occur inside the lens. Then at the exit one will see the divergent beam. However, for infinitely long CRL the processes of focusing and defocusing will be periodically repeated.

In this work we have perform for the first time testing of extremely long CRL, consisting of 407 elementary aluminum lenses. For this purpose the special holder was designed and manufactured. We have found that such a CRL allows to generate the submicron focus spot. The basic analytical expression for main properties of the long CRL, including focus distance, effective aperture, diffraction-limited size of focus spot, gain etc., is done in the next section. The computer program is elaborated for a calculation of these parameters as a function of energy. The experimental results are

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presented in the third section. The measurements were done for the photon energy E = 22, 25 and 30 keV. The measured values for the focus distance and aperture are in correspondence with the theory, although some small difference is found. The size of the focus spot and gain becomes larger than calculated values due to loss of resolution in the detector.

2. THEORY

Let us consider a simple experimental set-up shown schematically in Fig. 1. Here a long compound refractive lens is placed between the object plane and the detector (image) plane. The lens has a length L. The distances along the optical axis (axis z) are r_o (between the object and the front part of the lens) and r_i (between the exit part of the lens and the detector). The coherent source is a particular case of the object. The theory have to predict the transversal distribution of the wave field $E_i(x_i,y_i)$ at the detector plane from the known transversal distribution of the wave field $E_o(x_o,y_o)$ at the object plane and known parameters of the compound lens. In general case, the answer must be written as the integral relation $E_i(x_i,y_i) = \int dx_o dy_o\ G_t(x_i,y_i,x_o,y_o)E_o(x_o,y_o)$ for this fields where the propagator G_t has to take into account all parameters of the experimental set-up.

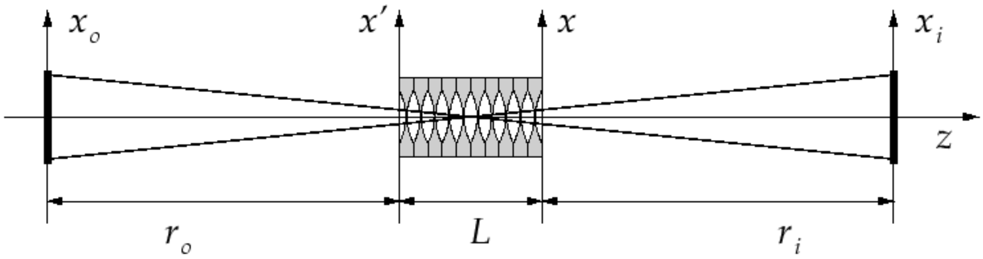

Fig. 1. Schematic view of the experimental set-up.

We are interested in the compound lens composed from parabolic elements. The parameters of such lens are shown in Fig. 2. Each elementary lens has a thickness p, a geometrical aperture a, and a thickness of the thin part d. Two parabolic surfaces of one element have a curvature radius R. In this particular case the transverse coordinates x and y are independent of each other and the total propagator $G_t(x_i,y_i,x_o,y_o) = \exp(-ik\eta dN)G(x_i,x_o)G(y_i,y_o)$. Here $k = 2\pi/\lambda$, λ is a wave length of x rays, $N = L/p$ is number of elements for the compound lens, $\eta = \delta - i\beta = 1 - n$, where n is a complex refractive index for the lens material. Therefore it is sufficient to consider the part of the total propagator $G(x_i,x_o)$ which describes propagation of the wave distribution along the x-axis.

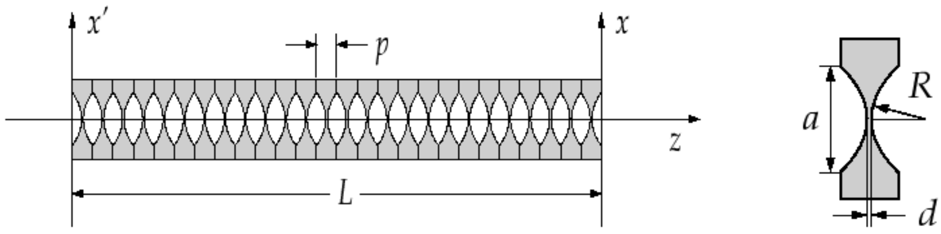

Fig. 2. Schematic view and main parameters of the long compound refractive lens

In the work [16] the analytical expression for the propagator $G(x_i,x_o)$ has been derived under a reasonable assumption that the ray trajectory is changed very slightly within one element of the compound lens. It seems to be a good approximation if the focal length for one element is much larger its thickness. Such assumption is fulfilled for 2D lenses because it is extremely difficult to realize a paraboloid of revolution with a very small curvature radius. This expression is as follows

$$G(x_i, x_o) = \frac{1}{(i\lambda r_g)^{1/2}} \exp\left(i \frac{\pi}{\lambda r_g} \left[g_i x_i^2 - 2x_i x_o + g_o x_o^2 \right] \right), \tag{1}$$

where

$$r_g = (r_i + r_o)c_L + \left(z_c - \frac{r_i r_o}{z_c} \right) s_L, \quad g_o = c_L - \frac{r_i}{z_c} s_L, \quad g_i = c_L - \frac{r_o}{z_c} s_L, \quad z_c = \left(\frac{pR}{2\eta} \right)^{\frac{1}{2}}. \tag{2}$$

Here we use a notation $s_L = \sin(L/z_c)$ and $c_L = \cos(L/z_c)$ for short.

The analytical formula for the propagator allows one to predict many of the properties of the compound refractive lens beyond the thin lens approximation. Below we present the analytical estimation for the main parameters of the lens. We note the propagator itself describes the image wave field, which is created by the lens for the point source. Since for hard x rays $\beta \ll \delta$, the absorption of x rays is negligible in estimation of many of the lens parameters. In some of them where the absorption is essential it is sufficient to use only first order correction over β/δ. Taking this into account we can introduce the critical length of the lens L_c, and the parameter $u_L = L/L_c$ which shows how long the lens is. The thin lens approximation is valid if $u_L \ll 1$. We introduce in addition the real values $S_L = \sin(L/L_c)$ and $C_L = \cos(L/L_c)$. The generalized lens formula is obtained from the condition $\mathrm{Re}(r_g) = 0$. This allows us to introduce the generalized focal length of the lens F_L and to define the distance r_{is} of imaging the point source behind the lens, which is located at the distance r_s in front of the lens. It is convenient to count both these distances from the middle of the lens. For these values the following expressions where obtained

$$L_c = \left(\frac{pR}{2\delta} \right)^{\frac{1}{2}}, \quad F_L = \frac{L_c}{S_L}, \quad r_{is} = \left(\frac{1}{F_L} - \frac{1}{r_s + b_L} \right)^{-1} - b_L, \quad b_L = \frac{L_c}{S_L}(1 - C_L) - \frac{1}{2}L \tag{3}$$

We note these formulae can be derived within geometrical optics [14] since we neglect absorption. In the limit of thin lens approximation $b_L = 0$ and one directly obtains the usual lens formula, whereas the generalized focal length F_L is replaced by $F = L_c^2/L$. This expression is well known in another form, namely, $F = R/(2N\delta)$. In general case, the focal length of the lens can be obtained as the value of r_{is} in the limit $r_s = \infty$. It is easy to calculate the first order correction to F. Since b_L is of order u_L^2, we have $F_L = F + L/6 + \cdots$. The diffraction theory, taking into account such correction, was developed in [17] .

Due to absorption the real aperture of the lens has no physical sense. Instead, one has to determine the effective aperture. For particular long compound lens the definition as FWHM (full width at half maximum) of the intensity profile just behind the lens is not correct because the beam can be partially focused inside the lens. We define the effective aperture A_{eff} as the integral intensity behind the lens for the plane incident wave of unit intensity. Another important characteristic is the diffraction limited focus size s_f which is closely related to the effective aperture. It is defined as FWHM of the intensity profile for the focus spot. For the plane incident wave we have from [16]

$$A_{eff} = \left(\frac{\lambda \delta F_L}{2\beta \alpha_L} \right)^{\frac{1}{2}}, \quad s_f = 0.47 \frac{\lambda F_L}{A_{eff}}, \quad \alpha_L = \frac{1}{2} \left(C_L + \frac{u_L}{S_L} \right) \tag{4}$$

However, the real focus size is always the image of the real source located at the finite distance from the lens. The more general formula for the diffraction limited focus size looks as more complicated

$$s_f = 0.47 \frac{\lambda (r_{is} + b_L)}{A_{eff}} K_L^{1/2}, \quad K_L = 1 - \frac{2(u_L - S_L)}{(u_L + S_L C_L)} \frac{F_L (b_L + L/2)}{(r_{is} + b_L)(r_s + b_L)} \tag{5}$$

The next important characteristic of the lens is the gain. We introduce first the ideal gain g for the point source and 1D.lens in the case $d = 0$. Such gain is the ratio of focused intensity integrated over the focus spot and the intensity without the lens integrated over the same region. In reality we need to take into account the real source size in both x and y directions and parasitic absorption on the thin parts of the lens. As a result, the real gain g_{2r} for 2D lens can be estimated as follows

$$g_{2r} = g^2 f_a m_x m_y, \quad g = 0.7614 \frac{A_{eff}}{s_f K_L^{1/2}} \frac{(r_s + r_{is})}{(r_s + b_L)}, \quad f_a = \exp(-2k\beta dN), \quad m_x = \frac{s_f}{\left(s_f^2 + (s_{sx} M)^2\right)^{1/2}} \tag{6}$$

Here s_{sx} and s_{sy} are the effective source size in *x*- and *y*- directions, the value M is the magnification factor as a ratio of source size projection and initial source size. In the case of the long compound lens this factor is defined by the formula $M = (r_{is} + b_L)/(r_s + b_L)$.

The formulae described above were used for elaborating the computer program, which calculates the main parameters of the long compound lens as a function of x-ray energy. The program was written in Java and can work in any operating system, including Pocket PC. The program can be downloaded from the internet [18]. The theoretical estimations shown below were taken as results of calculation by this program.

3. EXPERIMENT

The experiment was performed at the European Synchrotron Radiation Facility at the beam line BM5 (Fig. 3). The standard silicon –111 double crystal monochromator was used to select the energy of X-ray photons. The distance source-to-lens was r_s = 40 m. The effective transverse size of the source was 80 μm vertically and 250 μm horizontally. A FReLoN (Fast Readout Low Noise) CCD camera was used as a position sensitive detector with 2048x2048 pixels and pixel size 0.67x0.67 μm^2.

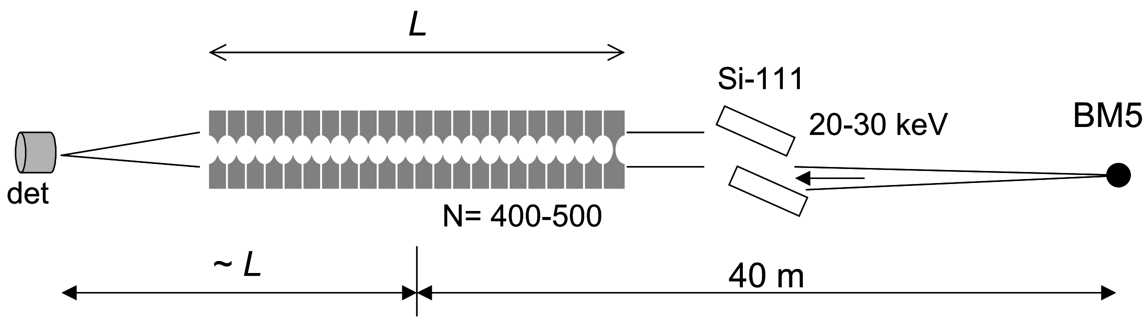

Fig. 3. Experimental setup

The compound refractive lens is composed from the standard Al parabolic lenses made by RWTH in Aachen [7-10,19] from aluminium. The parameters for one element are R = 0.2 mm, p = 1 mm, d = 15 μm. The compound lens has 407 elements. Its longitudinal length is *L* = 40.7 cm. The special holder was manufactured for the first time to accommodate such number of elements (Fig. 4).

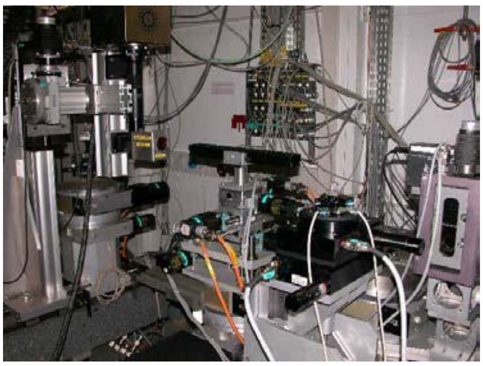

Fig.4. Photos of the experimental setup (left) and the lens holder (right)

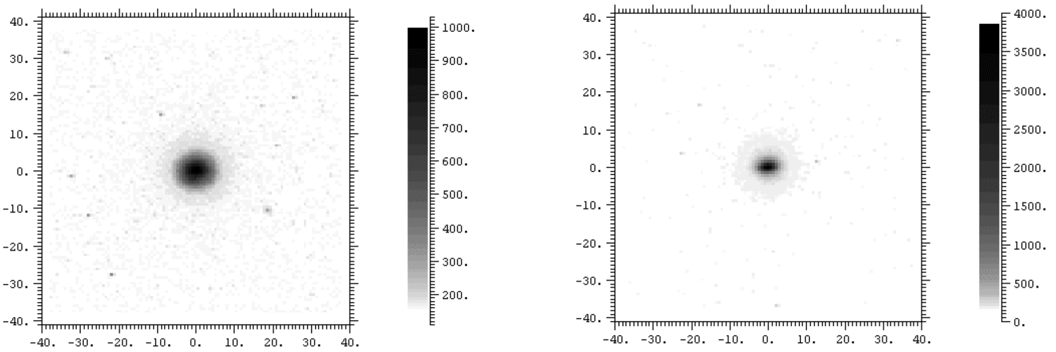

Fig. 5. The images of the beam at the distance of focusing (right) and at the distance 2.5 cm shorter (left). E = 22 keV. The transverse distances are measured in microns. The scale at the right shows the intensities in arbitrary units.

The measurements were made for energies E = **22**, **25** and **30** keV. For all these energies the distance of focusing was obtained as a distance r_{ise} for optimal focusing, counted from the middle of the lens. We found the values r_{ise}. = **29**, **37** and **51** cm for the energies pointed above. Fig. 5 shows the images of the beam for E = 22 keV recorded at the focus distance (right) and at the distance 2.5 cm shorter (left). One can see the left image is round due to circular symmetry of the lens aperture. The source size does not influence it. The right image has ellipsoidal shape due to a symmetry of the source projection. However, a low resolution of the FReLoN camera can not reveal the vertical size of the source projection. The theoretical estimation for the source image distance (focusing), made along the above formulae, gives values r_{is} = **27**, **34** and **47** cm correspondingly. The thin lens approximation gives for this distance values **21**, **29** and **41** cm for the same energies. Finally, the thin lens approximation corrected by L/6 gives the values **29**, **36** and **48** cm.

It is easy to see that the thin lens approximation is certainly not valid for our lens. As for the L/6 correction, it is close to the accurate values and is slightly larger. The difference increases with decreasing the energy. However, the experimental values become larger than the L/6 correction. And the difference increases with increasing the energies. The reason of such small difference is not clear. One of the reason may be a small deviation of the surface shape of the real elementary lens from the parabola in such a way that the curvature radius at the vertex is larger than at the tails. This difference may be negligible for small number of elements but becomes apparent for large number. However, additional measurements and computer simulations are needed to understand the real situation.

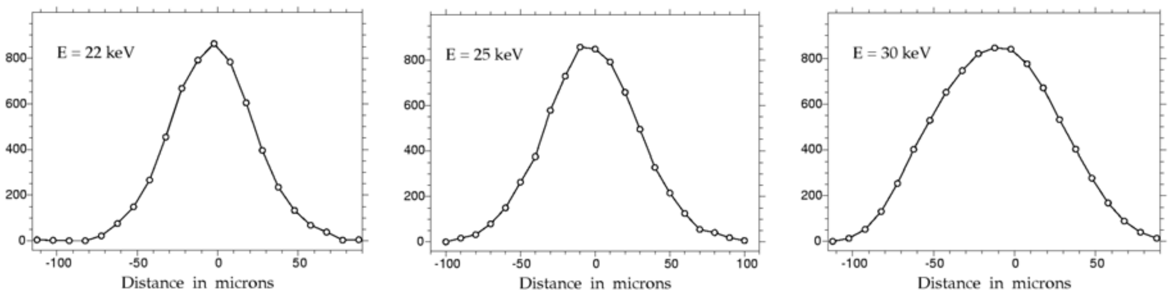

Fig. 6. Integral intensity (arb. units) behind the lens for the beam limited by a slit of 10x20 μm^2 size at various slit positions.

To evaluate the effective aperture of the compound lens, we performed measurements of the integral intensity behind the lens under conditions where the incident wave front was limited by small rectangular slit of 10x20 μm^2 size. The slit was scanned across the lens aperture. In this way we obtain quasi-gauss distribution of the intensity for each energy. The results of measurement are shown in Fig. 6, where the intensity is shown in arbitrary units. The FWHM of these profiles should correspond approximately to the effective aperture A_{exp}. We found that for energies E = 22, 25 and 30 keV the measured effective apertures are A_{exp} = **70**, **80** and **100** μm. These values can be compared with the theoretical estimations of A_{eff} along the formula (4). The program calculates A_{eff} = **65**, **73** and **89** µm. We should note that the theoretical values are obtained as an integral of the gauss function but not from FWHM. However, FWHM for the gauss function can be calculated from the integral value as a result of a simple multiplication by a factor 0.94, and this results in a good agreemebnt between experimental and calculated data.

It is of interest to evaluate the depth of focus for such lens. Fig. 7 shows the distribution of the intensity maximum along the optical axis near the focus. Intensity is measured in arbitrary units. The increasing of the intensity with X-ray energy occurs due to the reduction of the absorption in the thin part of elementary lenses. One can see also increasing of the focus depth with the energy and this correlates with increasing of the focus distance.

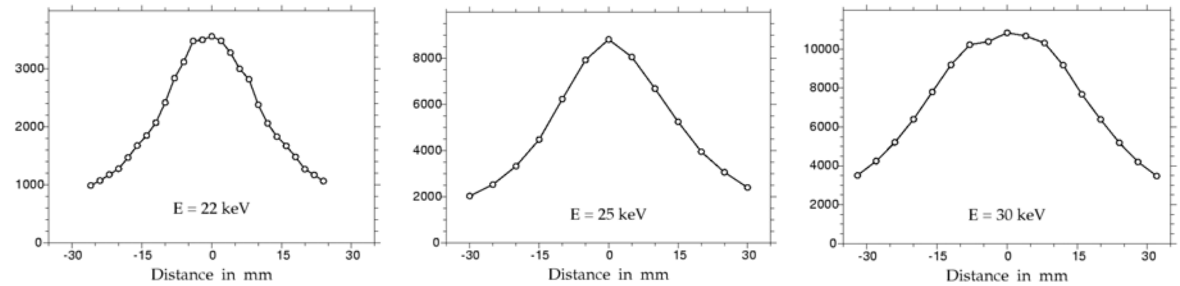

Fig. 7. Intensity distribution along the optical axis showing the lens depth of focus in arb. units.

Since the compound lens is very long it is obvious that optimal angular position is very important for efficient focusing. The integral intensity as a function of rotating angle (theta and phi) around axes perpendicular to the optical axis is shown in Fig. 8 for E = 25 keV. Such curves can be called as “rocking curve” similarly to the rocking curves measured for the single crystal Bragg diffraction. We have found that the angular width W_{exp} of the compound lens is equal to **0.4** mrad in both rotating scans. Introducing an angular acceptance of the lens as $W = A_{eff}/L$, we calculate W = **0.18** mrad. for E = 25 keV These value is more than twice smaller than W_{exp} . Thus, this simple analysis leads to a direct verification of the fact that the ray trajectories inside the lens are not straight.

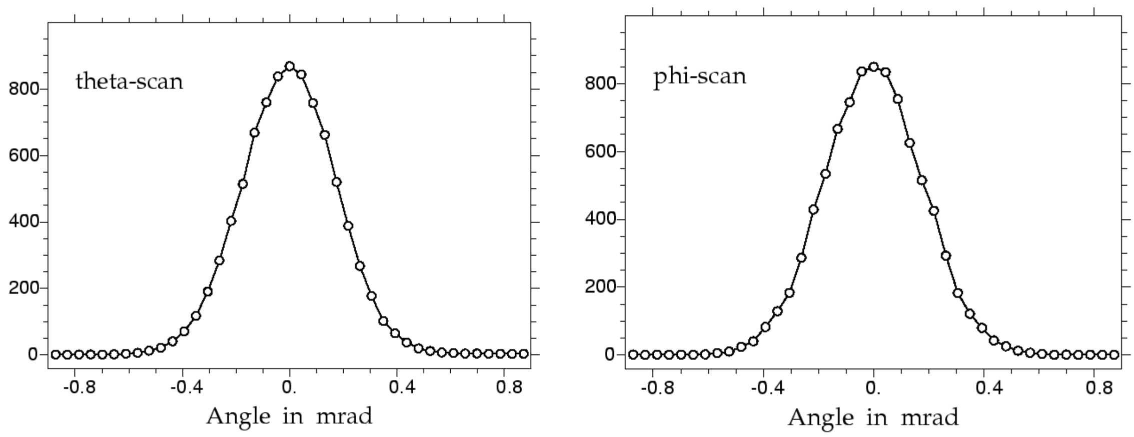

Fig. 8. “Rocking curves” for the long compound lens at E = 25 keV. The integral intensity is measured in arb. units

4. DISCUSSION

It is known that modern planar technology allows one to create the parabolic profile with the curvature radius as small as 1 μm. However, it is not the case for 2D parabolic lens. Therefore the usage of a compound lens consisting of very many elements is a unique way to decrease effectively the curvature radius of the parabolic surface. For example, our lens of 407 elements with R = 0.2 mm is equivalent to one single-concave lens with the curvature radius 0.49 μm. In addition, it is very difficult to make one parabolic surface without shape errors. On the other hand, errors in parabolic profiles of elementary lenses can compensate each other. It is certainly the advantage of compound lens.

The main disadvantages of the thick compound lens are its long length and additional parasitic absorption in the thin parts of elementary lenses, which leads to a essential gain decrease. As it follows from the theory [15-17] the optimum lens length is equal to the focus distance counted from the end of the lens or 2/3 of the focus distance counted from the middle point. The following increasing the number of elementary lenses decrease only slightly a total focus distance including the lens length itself. As for the absorption, the factor $f_a = \exp(-2k\beta dN)$ decreases very sharply with decreasing the photon energy. Therefore there is a certain lower limit on the X-ray energy region where the long compound lens can be use. For example, for our lens the theoretical gain value increases by factor three when the energy is changed from 22 to 25 keV and then from 25 to 30 keV. Of course, making the elementary lens with extremely thin dam is very promising. We have to note that the magnification factor for the thick lens is not the same as for thin lens. Therefore a special study of imaging with magnification must be performed.

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