Theoretical Analysis of X-ray Compound Refractive Lens Optical Properties

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Abstract. We present a theoretical analysis of optical properties of parabolic compound refractive lenses (PCRL) used for x-ray focusing and micro-imaging. The PCRL with a large number of elements can be considered as a parabolic medium along the x-ray path. The problem of x-ray coherent wave propagation inside such a medium is solved exactly. The analytical formula is obtained for the PCRL imaging propagator as a parabolic wave with complex parameters due to absorption of x rays inside the lens. The fast and universal computer program is developed for simulating the images obtained with PCRL. An example of model object imaging is presented and discussed in details. The model object is a round hole of 3 μm diameter inside the silicon plate of 3 μm thickness. The main optical parameters of PCRL such as an effective aperture, a diffraction limited linear size of focus spot and a focal distance are calculated analytically and discussed. The PCRL has no aberration while a long single plano-concave lens or bi-concave lens have.

INTRODUCTION

After a first successful attempt [1] to apply small x-ray refraction for focusing a synchrotron radiation beam, the x-ray refractive optics was developed extensively in recent years. The first parabolic lenses with a relatively large aperture were compound, i.e. they consisted of many individual lenses packed closely together [2], each of them had sufficiently large curvature radius R. A fabrication of single lens with extremely small curvature radius of parabolic profile was also demonstrated [3] and developed in further works. We note both the compound lens and the single lens with the same focal length F and geometrical aperture A have the same longitudinal size L if $L << F$, and differ only by the value of R (see Fig. 1) The compound refractive lenses are easy to fabricate. They used now for both x-ray focusing and x-ray micro-imaging [4] with high resolution. As for the theory, the thin lens approximation of visible light optics can be used when $L << F$ for both the single lens and the compound lens. Some generalization is necessary due to the fact that the x-ray lenses are always absorbing. This leads to the new phenomenon of visualization of the transparent objects even under the conditions of focused image [5]. The significant difference appears for the rather long lenses when L is comparable with F. The exact theory of coherent x-ray wave propagation through the long compound refractive lens with very many elements (bottom lens at the Fig. 1) was developed recently [6,7]. In this work we point out advantages of the compound refractive lens compared to the single lens in the case when L is comparable or even larger F.

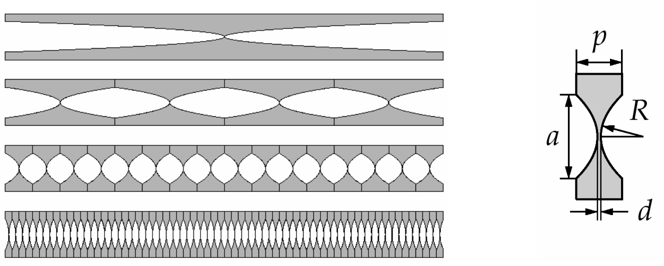

FIGURE 1. Various x-ray refractive lenses with the same geometrical aperture and length, which have the same focal length of thin lens approximation (left) and parameters of one element (right).

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PROPAGATION OF COHERENT X-RAY WAVE THROUGH PARABOLIC MEDIUM

Because of very small wave length λ and refraction, the paraxial approximation can be applied in the theory of x-ray optics with a good accuracy. So the wave emitted by the point source can be considered as a parabolic wave having the parabolic profile of the phase front. The free space propagator, describing a transfer of the coherent wave field through the empty space along the optical axis (z-axis). is also a parabolic wave. Consider the compound lens consisting of very many thin elements with a relatively large curvature radius. Since the trajectory of rays inside one element is changed negligibly, the element can be formally replaced by one which is homogeneous longitudinally and has parabolic variation of optical density transversely. A stack of N such elements looks like a parabolic medium along the ray path with the complex refractive index $n(x,y) = 1 - \delta s(x,y) + i\beta s(x,y)$ where δ is a decrement of refractive index, β is an index of absorption of the lens material, and $s(x,y) = d/p + (x^2 + y^2)/2pR$. The parameters d, p and R are shown on Fig.1. As it was shown in [6], the parabolic wave field remains parabolic during a propagation through a parabolic medium. All what happens is a change of a wave front curvature.

In the case of planar lens only one of two transverse coordinates is of interest. In the case of parabolic lens with round aperture each of two transverse coordinate can be considered separately. For this reason we consider here the case of one transverse coordinate. It is evident that the propagator for more complicated case, including a distance r_o from the object to the front side of the lens and a distance r_i from the back side of the lens and the image detector, is a parabolic wave as well. Such a propagator was calculated in [7] as

$$G(x_i, x_o) = \frac{1}{(i\lambda r_g)^{1/2}} \exp\left(i\frac{\pi}{\lambda r_g}\left[g_i x_i^2 - 2x_i x_o + g_o x_o^2 \right] \right), \quad (1)$$

where x_o and x_i are transverse coordinates at the plane just behind the object with the longitudinal coordinate z_o and image plane with the coordinate z_i respectively, and

$$r_g = (r_i + r_o)c_L + \left(z_c - \frac{r_i r_o}{z_c} \right) s_L,\ g_o = c_L - \frac{r_i}{z_c} s_L,\ g_i = c_L - \frac{r_o}{z_c} s_L \quad (2)$$

Here

$$z_c = \left(\frac{pR}{2\eta} \right)^{\frac{1}{2}},\ c_L = \cos\left(\frac{L}{z_c} \right),\ s_L = \sin\left(\frac{L}{z_c} \right),\ L = pN,\ \eta = \delta - i\beta. \quad (3)$$

We assume that the total radiation wave is written as $E(x,y,z) = \exp(ikz)A(x,y,z)$ where $k = 2\pi/\lambda$ and $A(x,y,z)$ describes the propagation of transverse x-ray wave properties. The propagator allows one to calculate $A_i(x,y,) = A(x,y,z_i)$ from the known wave field $A_o(x,y) = A(x,y,z_o)$as

$$A_i(x_i, y_i) = \exp(-ik\eta dN) \int dx_o dy_o G(x_i, x_o) G(y_i, y_o) A_o(x_o, y_o) \quad (4)$$

On the other hand, the known wave field $A_o(x,y)$ can be written as a product of an incident wave and a transmission function for the object.

COMPUTER SIMULATION OF IMAGES

The formulae (1) – (4) were used for creating the fast and universal computer program for simulating the images obtained with parabolic compound refractive lenses. The program was written within the Igor-Pro environment [8]. It works for any combination of parameters. The case of focused image when $\mathrm{Re}(r_g) = 0$ is of most interest. As an example, Fig. 2 shows calculated images of a model object which is a round hole of 3 μm diameter inside a thin silicon plate of 3 μm thickness. The plate is perpendicular to the optical axis. Therefore at the border of the hole the x-ray wave phase jumps on the value $\Delta\varphi = -0.294$ when it passes from the hole to the silicon, whereas an amplitude decreases on the factor exp(–0.001), i.e. negligibly. The calculation was done for the x-ray energy 25 keV. The compound lens has the following parameters: material Al, $N = 100$, $R = 0.2$ mm, $p = 1$ mm, $d = 0.01$ mm. Such parameters correspond to the real lens of Lengeler's group [9]. It is assumed that the point source is placed on the optical axis at the distance $r_s = 40$ m from the object, the distances $r_o = r_i = 2.2973$ m satisfy the condition of imaging without a magnification. In this case the lens's critical length $L_c = \mathrm{Re}(z_c) = 0.34$ m, the thin lens focal length $F = L_c^2/L = 1.157$ m.

Fig. 2 shows a ratio G of the image intensity to the intensity of the incident wave at the same place. The contrast does not take into account a background, i.e. the black corresponds to G_{min} and the white corresponds to G_{max}. The four panels are correspondent to various positions of the object relative to the lens. The panel (a) shows the image of the object placed at the center of the lens aperture. On the other panels the object is shifted on 50 μm to the left (b), on 50 μm to the top (c) and on 50 μm to the left and top simultaneously (d). The panels have different contrast which is not realized on the figure. So G_{min} = 0.788, G_{max} = 0.802 at the panel (a), G_{min} = 0.362, G_{max} = 0.505 at the panels (b) and (c), G_{min} = 0.177, G_{max} = 0.302 at the panel (d), whereas the linear size of the panels is 6 μm.

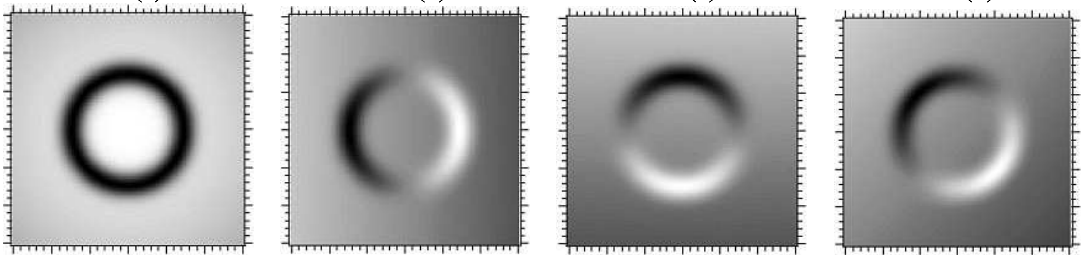

FIGURE 2. Computer simulation of images of small hole inside the silicon plate, obtained with compound refractive lens. Panels (a) to (d) correspond various positions of the hole relative to the lens center. See text for details.

Let us discuss the properties of images. First of all, the images are clear, they don't contain artifacts. This is because the lens has no aberrations. This fact follows straightforwardly from the analytical formula for the propagator. When the generalized lens formula $\mathrm{Re}(r_g) = 0$ is fulfilled, the propagator modulus function becomes like a very strong peak. On the other hand, the central part of hole is not visible because the absorption inside a thin silicon plate is very small. The absorption reveals itself only on the panel (a) where a contrast is very small. Meantime the border of the hole is visible very well, and the contrast grows with increasing a distance between an object location and a lens center. The reason of such contrast was discussed in [5]. Due to finite resolution of the propagator even a sharp phase jump can be seen after a calculation of the integral in (4). The thickness of the border image shows the diffraction limited size of the focus spot which depends on the effective aperture of the PCRL due to absorption.

We note the border image can show both the phase jump and its sign. The contrast is homogeneous along the border for the central location and it is heterogeneous when the object is moved from the center. As was pointed out shortly in [7] this peculiarity can be explained in terms of geometrical optics. Indeed, the phase gradient cannot change the intensity of the ray, but it can change a trajectory of the ray path. In the case of focused image all the rays emitted from one point at the object go to the same point at the image. In reality all trajectories begins at the source. If the transparent object is phase homogeneous, the trajectories don't disturbed by the object. The image shows only the lens aperture. If the object produces the phase gradient at some region, the trajectory of rays passing this region is changed. As a result the ray goes through the lens in other place where absorption by the lens is different. Finally, the ray come to the same point at the image, but the intensity of the ray will be changed. Let us consider, for example, the image of the panel (b). The left part of border is black because the ray was deviated to outer part of lens aperture where the absorption inside the lens is larger. The right part of border is white due to an opposite sign of deviation angle. As for the top and bottom parts of border, they does not show contrast because the rays were deviated in the parts of the lens aperture which have the same thickness and produce the same absorption. Similar analysis allows one to understand the images of panels (c) and (d).

ESTIMATION OF THE LENS OPTICAL PROPERTIES

The analytical formula for the image propagator allows estimation of many properties of the PCRL in general form. The main parameters are the focal length, the effective aperture and the focus size in the case of focusing a plane wave. The result has to be obtained by integration of (1) over a coordinate x_o. However, the same result can be obtained from the propagator itself considering the point object at the optical axis which is moved on infinite distance $r_o = \infty$ and dividing the propagator on the amplitude of spherical wave for such distance. Thus, the focal length is determined from the condition of finite $\mathrm{Re}(r_g)$ when r_o tends to infinity. Since $\delta >> \beta$ we can neglect β in such a calculation. Then from (2) we calculate the focal length of the PCRL counted from the back side of the lens as $F_b = L_c/\tan(L/L_c)$. In a thin lens approximation when $L << L_c$ we arrive to $F = L_c^2/L$. It is easy to calculate the next correction to the thin lens approximation. When the focal length is counted from the middle of the lens we obtain $F_c = F_b + L/2 \approx F + L/6$. Such a correction is rather useful in many practical cases. For example, in the case of preceding section the distance of imaging is well approximated as $r_i = 2F_c - L/2$. It is useful also to define a generalized focal length as $F_L = L_c/\sin(L/L_c)$ which comes to F_c when $L << L_c$.

According to geometrical optics the distance F_c is the distance where all rays within the aperture, which are parallel to the optical axis before the lens, intersect the optical axis after the lens. We obtain the same distance for all the rays, therefore there is no aberration. It is useful to compare the compound lens with the single plano-concave lens. Let F_c be a distance where the rays at the central part of the aperture intersect the optical axis and F_e be the same for the rays at the edge of aperture. If the back side of plano-concave lens is concave, we obtain $F_c = F - L/2$, $F_e = F + L/2$. Therefore the aberration is large for the long lens. The same analysis for a bi-concave lens in the case of $L << F$ leads to $F_c = F$, $F_e = F + L/4$. Now the aberration is significantly smaller. The calculation can be made for two, three and more bi-concave lenses. We calculated $F_c = F + L/8$ for two lenses, $F_c = F + (4/27)L$ for three lenses. Hence the limit value $F_c = F + L/6$ of the case of very many bi-concave lenses is reached rather quickly.

As it follows from the analytical expression for the propagator, the intensity distribution at all distances after the PCRL is described by Gaussian. At the distance of plane wave focusing $r_i = F_b$ the Gaussian has a minimum full width at half maximum (FWHM) which can be treated as a diffraction limited linear size of the focus spot s_γ. The effective aperture of the lens is determined by absorption of x rays in the lens material. We define a diameter of the effective aperture A_γ as the integral intensity of radiation passed through the lens. This value is the same for all distances from the back side of the lens, therefore we can calculate A_γ at the distance of plane wave focusing. Due to restricted volume we present only the result for $L \leq L_c$ as

$$A_\gamma = \int dx_i |A_i(x_i)|^2 = \left(\frac{\lambda F_L}{2\gamma\alpha_L}\right)^{\frac{1}{2}},\ \alpha_L = \frac{1}{2}\left(C_L + \frac{L}{L_c S_L}\right),\ s_\gamma = 0.47\frac{\lambda F_L}{A_\gamma},\ \gamma = \frac{\beta}{\delta}. \qquad (5)$$

Here $C_L = \cos(L/L_c)$, $S_L = \sin(L/L_c)$ and we applied a linear over a small parameter γ approximation. We note s_γ is proportional to λ and F_L and inverse proportional to A_γ according to general optical law. However, the parabolic absorbing lens differs from the lens of visible light optics. Here the sizes of real lens and focus spot are slightly larger than A_γ and s_γ due to apparent tails of Gaussian distribution. This results in appearance of the numerical coefficient 0.47. On the other hand, the aperture is proportional to square root of λF_L. As a result, the linear size of focus spot is proportional to the aperture for all values of PCRL length with the coefficient $0.94\alpha_L\gamma$ which depends only slightly on L. A mean value of coefficient is 0.8γ.

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REFERENCES

1. Snigirev, A., Kohn, V., Snigireva, I., and Lengeler, B., Nature, 384, 49-51 (1996).
2. Lengeler, B, Schroer, C. G., Richwin, M., Tummler, J., Drakopoulos, M., Snigirev, A., and Snigireva I., Appl. Phys. Lett., 74, 3924-3926 (1999).
3. Aristov, V. V., Starkov, V. V., Shabelnikov, L. G., Kuznetsov, S. M., Ushakova, A. P., Grigoriev, M. G., and Tseitlin, V. M., Opt. Comm., 161, 203-208, (1999).
4. Schroer, C. G., Benner, B., Gunzler, T. F., Kuhlmann, M., Zimprich, C., Lengeler, B., Rau, C., Weitkamp, T., Snigirev, A., Snigireva, I., and Appenzeller J., Rev. Sci. Instrum., 73, 1640-1642 (2002).
5. Kohn, V., Snigireva, I., Snigirev A., Opt. Comm., 216, 247-260 (2003).
6. Kohn, V. G., JETP letters, 76, 600-603 (2002).
7. Kohn, V. G., JETP, 97, 204-215, (2003).
8. Internet web-site "http://wavemetrics.com".
9. Lengeler, B., Schroer, C.G., Benner, B., Gerhardus, A., Gunzler, T.F., Kuhlmann, M., Meyer, J., Zimprich, C., J. Synchr. Rad., 9, 119-124 (2002).