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On the coherence length of x rays

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It is well known that in order to obtain an interference pattern from two sources it is essential that the radiation from these sources should be coherent. In order to obtain two coherent beams in optics it is usual to separate the light from a single source by means of lenses or mirrors and then to reunite the two beam after each of them has traveled over a different optical path. An interference pattern will even then only be formed if the path difference between the beams is shorter than the so called coherence length $L = c\tau$, where c is the velocity of light and τ is the lifetime of the exited state of the radiating atom.

Since in the case of x rays reflection and refraction are negligibly small, in an analogous interferometer for x rays, instead of mirrors and lenses use is made of diffraction scattering in an ideally periodic crystal. It should be note that the interference patterns associated with diffraction in an ideal crystal having a weak absorption appear quite sharply, and have long been known as pendulum bands. However, only quite recently has it been suggested that the coherence length L (length of train) should be measured experimentally by reference to the vanishing of the bands.

It is the aim of this note to show that in such experiments it is not the length of the trains but only the degree of deviation of the radiation from monochromatic which may be measured, i.e., the frequency spread of the radiation intensity. At the instant of time t_0, let the atom emit a wave with frequency ω_0, the amplitude of which diminishes to a major extent in time τ

$$E(t) = \theta(t - t_0)\exp(i\omega_0 t) f([t - t_0]/\tau) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \exp(i\omega t) F(\omega - \omega_0) \quad (1)$$

where

$$F(\omega) = \exp(-i\omega t_0) E(\omega), \quad E(\omega) = \int_0^{\infty} dt \exp(-i\omega t) f(t/\tau), \quad (2)$$

For example, in the case of exponential damping

$$f(x) = \exp(-x), \quad E(\omega) = 1/(i\omega + 1/\tau), \quad (3)$$

Each ω harmonic, interacting with the instrument, is split into two components, which are then added together again. Furthermore, both the amplitudes of these waves and the phase difference between them may, generally speaking, alter

$$\exp(i\omega t) \to (R_1(\omega) + R_2(\omega)\exp(i\varphi(\omega)))\exp(i\omega t), \tag{4}$$

Let us now substitute the right-hand side of (4) into (1) and calculate the square of the modulus E(t)

$$\begin{aligned} I(t) = \int \frac{d\omega}{2\pi}\int \frac{d\omega'}{2\pi}\exp[i(\omega-\omega')(t-t_0)]E(\omega-\omega_0)E(\omega'-\omega_0) \\ \times[R_1(\omega)+R_2(\omega)\exp(i\varphi(\omega))][R_1^*(\omega')+R_2^*(\omega')\exp(-i\varphi(\omega'))], \end{aligned} \tag{5}$$

Equation (5) describes the intensity of the radiation which will be recorded by the detector from one train. However, during the period of observation radiation from many atoms will fall into the detector, the instants of excitation of these atoms not being precisely established. Hence Eq.(5) should be averaged with respect to t_0. As a result of averaging we obtain

$$I(t) = \int \frac{d\omega}{2\pi}\left|E(\omega-\omega_0)^2\right|(\left|R_1(\omega)\right|^2 + \left|R_2(\omega)\right|^2 + 2\,\mathrm{Re}[R_1^*(\omega)R_2(\omega)\exp(i\varphi(\omega))]), \tag{6}$$

The last term in Eq.(6) describes the interference pattern. Let the path difference be equal to l. Then the phase difference $\varphi = \omega\, l/c$. If $\Delta\omega$ is the characteristic range of integration in (6), the interference pattern will only be sharp if $l \ll 2\pi c/\Delta\omega$.

However, it is clear from Eq.(6) that $\Delta\omega$ is determined, first, by the width of the spectral line of the source measured in terms of intensity, i.e., as if the individual monochromatic waves were incoherent. We may envisage a situation in which two or more sources of completely different natures create a wave packet having an energy width close to $\hbar/\tau$. In this case the interference pattern is not connected with the length of the train, and yet it is indistinguishable from that just considered. Second, it is essential to allow for the spectral width of the instrument. Thus in the case of two-wave x-ray diffraction in which a wave of frequency ω_0 is incident at the Bragg angle, the reflected beam only has a appreciable intensity in a narrow frequency range $\Delta\omega \approx 10^{-5}\omega_0$, which generally speaking is smaller than the spectral width of the characteristic radiation of x-ray tube.

Translated by G. D. Archard